# Homework Assignment (Problem Set) 4:

Note, Problem Set 3 directly focuses on Modules 7 and 8: Metaheuristic Algorithms and Monte Carlo Simulation

***4 Questions***

Rubric:

All questions worth 37.5 points

37.5 Points: Answer and solution are fully correct and detailed professionally.

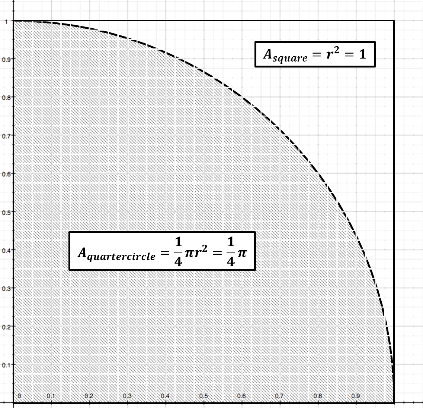
25-37 Points: Answer and solution are deficient in some manner but mostly correct.

15-24 Points: Answer and solution are missing a key element or two.

1-14 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.

0 Points: No answer provided.

**1.** Perform Monte Carlo integration using R statistical programming or Python programming to estimate the value of π. To summarize the approach, consider the unit quarter circle illustrated in the figure below:



Generate pairs of uniform random numbers, where and , and each pair represents a point in the unit square. To obtain an estimate of π, count the fraction of points that fall inside the unit quarter circle and multiply by 4. Note that the fraction of points that fall inside the quarter circle should tend to the ratio between the area of the unit quarter circle (i.e., ¼) as compared to area of the unit square (i.e., 1). We proceed step-by-step:

1. Create a function insidecircle that takes two inputs between 0 and 1 and returns 1 if these points fall within the unit circle.

Box A- columns x and y take on random numbers from 0 to 1 “=RAND()”

1. Create a function estimatepi that takes a single input , generates pairs of uniform random numbers and uses insidecircle to produce an estimate of as described above. In addition to the estimate of , estimatepi should also return the standard error of this estimate, and a 95% confidence interval for the estimate.

Box B- column N indicates how many pairs of uniform random numbers are used to estimate pi.

Column estimatepi is how many points are in the circle \* 4.

Column SE is the standard error “=STDEV([points])/SQRT(COUNT([points]))\*4”

Column 95%CI is how much to add/subtract to estimatepi for confidence intervals

“=CONFIDENCE(0.05,4\*STDEV.P([points]),COUNT([points]))”

1. Use estimatepi to estimate for = 1000 to 10000 in increments of 500 and record the estimate, its standard error and the upper and lower bounds of the 95% CI. How large must be in order to ensure that your estimate of is within 0.1 of the true value? In other words, how large must be in order to ensure that the confidence interval is within 0.1 of the estimate of ?

Box C- column 95%CI shows for all N>=1500, we are 95% confident that the CI is within 0.1 of estimatepi

1. Using the value of you determined in part c), run estimatepi 500 times and collect 500 different estimates of . Produce a histogram of the estimates and note the shape of this distribution. Calculate the standard deviation of the estimates – does it match the standard error you obtained in part c)? What percentage of the estimates lies within the 95% CI you obtained in part c)?

In Box D, 500 replications of estimatepi (N=1500), are placed in a histogram which is distributed as approximately normal. SD of estimates (0.0419) is aprox. equal to SE of part C (0.0427) and 93.2% of estimates lie within the CI of part C.

D

B, C

A

**Graphical user interface, application, table, Excel

Description automatically generated**

**2.** A salesperson in a large bicycle shop is paid a bonus if he sells more than 4 bicycles a day. The probability of selling more than 4 bicycles a day is only 0.40. If the number of bicycles sold is greater than 4, the distribution of sales as shown below. The shop has four different models of bicycles. The amount of the bonus paid out varies by type. The bonus for model A is $10; 40% of the bicycles sold are of this type. Model B accounts for 35% of the sales and pays a bonus of $15. Model C has a bonus rating of $20 and makes up 20% of the sales. Finally, a model D pays a bonus of $25 for each sale but accounts for only 5% of the sales. Develop a simulation model to calculate the bonus a salesperson can expect in a day. Note that if the salesman sells more than 4 bikes, he earns a bonus on each bike sold. That is, if 6 bikes are sold, he earns a bonus on each of the 6 bikes, not just the 2 bikes beyond the 4 bike cutoff.

Table

Number of Bicycles Sold Probability

5 0.35

6 0.45

7 0.15

8 0.05

**Graphical user interface, application, table, Excel

Description automatically generated**

I first set up the tables with weighted probabilities – one for the number of bikes sold, and one for the type of bikes sold. Using random numbers, I simulate how many bikes are sold, and assign each bike to a type also using random numbers (and taking in to account the weighted probabilities).

In the middle table, I ran 1000 simulations of how many bikes are sold and of what type to calculate the bonus. 4 or less results in no bonus whereas 5+ can be calculated from the bonus values.

In the right table, I replicate the simulation of N=1000, 100 times and plotted the average bonus amount in a histogram which shows we can expect $33-$34 of bonus on average.

**3.** Michael is 24 years old and has a 401(k) plan through his employer, a large financial institution. His company matches 50% of his contributions up to 6% of his salary. He currently contributes the maximum amount he can (i.e., 6%). In his 401(k), he has three funds. Investment A is a large-cap index fund, which has had an average annual growth over the past 10 years of 6.63% with a standard deviation of 13.46%. Investment B is a mid-cap index fund with a 10-year average annual growth of 9.89% and a standard deviation of 15.28%. Finally, Investment C is a small-cap Index fund with a 10-year average annual growth rate of 8.55% and a standard deviation of 16.90%. Fifty percent of his contribution is directed to Investment A, 25% to Investment B, and 25% to Investment C. His current salary is $48,000 and based on a compensation survey of financial institutions, he expects an average raise of 2.7% with a standard deviation of 0.4% each year. Develop a simulation model to predict his 401(k) balance at age 60.

**Graphical user interface, application, table, Excel

Description automatically generated**

I first set up the means and standard deviation tables on the left – one for starting salary/raises, and one for average and standard deviation of investments by type.

In the middle table, I simulated years 24-60. The salary raise increases using a random number along with the normal distribution of the average raise and standard deviation of raise. 6% is invested plus an additional 3% is matches, so yearly contributions = 9% of salary.

Contribution percentages are firm, but the return on investment will vary between funds. For example, year 30/investment A is 50% of the yearly contributions (9% of year 30 salary) + current funds (year 29/investment A) + growth from last year (year 29/investment A times growth factor – a random number of avg growth for A and sd of A).

I simulated the 401k balance at age 60, 100 times which are shown in the histogram. The average balance at age 60 is $1,206,864 and the median balance is $1,080,390.

**4.** Develop a simulated annealing procedure in either R or Python to solve the following knapsack problem: (Note, this problem can be solved to optimality using integer programming; however, the focus of this question is on developing the simulated annealing method).

Simulated annealing:

Maximize 12x1 + 16x2 + 22x3 + 8x4  
S.T. 4x1 + 5x2 + 7x3 + 3x4 ≤ 14  
 xi ~ binary for all i

A screenshot of a computer

Description automatically generated with medium confidence

Double check with pulp solutions:

Text

Description automatically generated with medium confidence

Both methods come to the solution of taking x1, x3, and x4, for a total value of 42 and total space of 14.